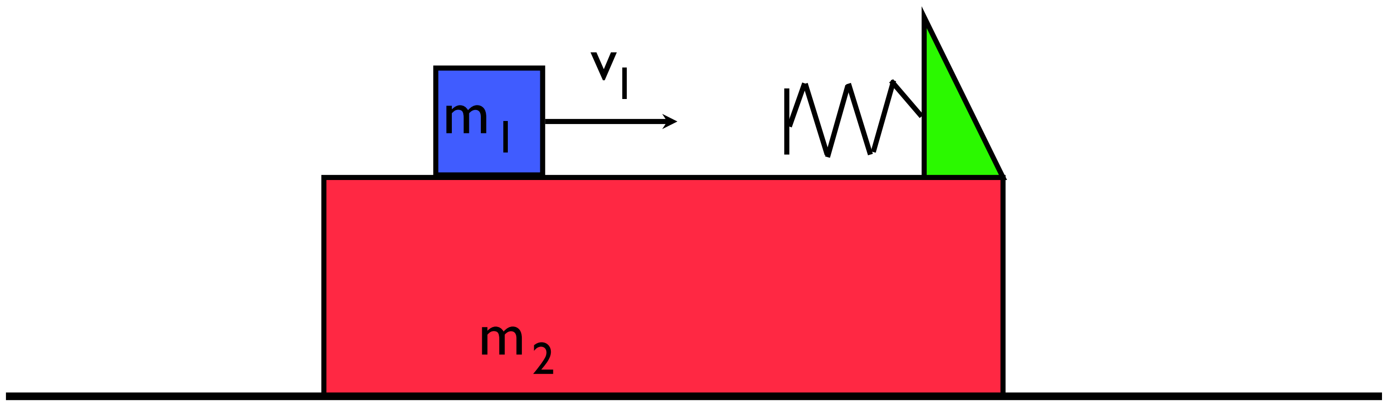
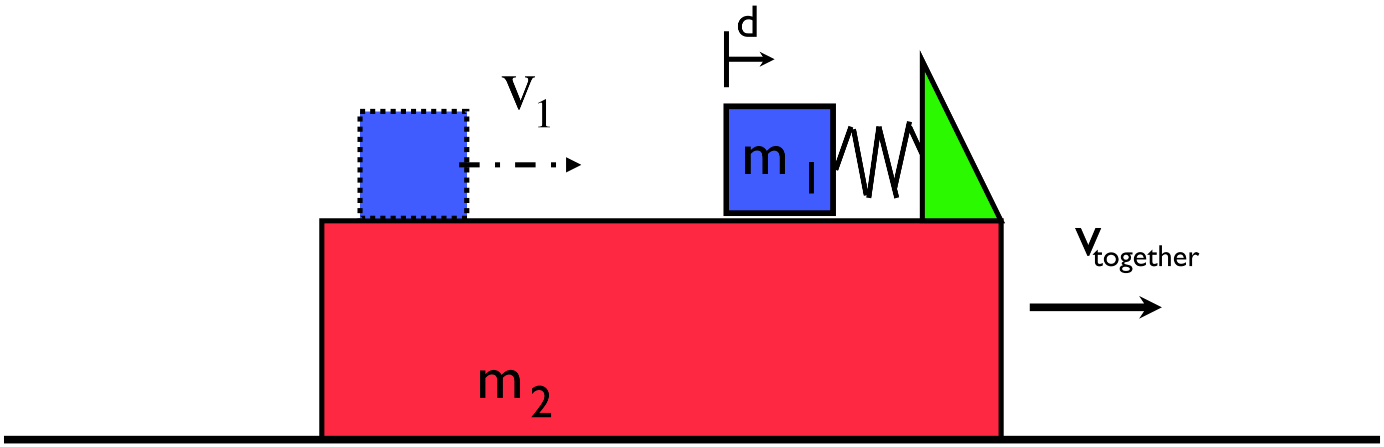
ONE OFF-THE-WALL QUESTIONS

1.) Why is it not a good idea to have the mass of a bullet be the same as the mass of the gun that fires it? (video?)

A Collision Problem w/Spring

A mass m_2 sits stationary on a frictionless tabletop. Sitting on that mass is a second mass m_1 that is moving to the right with velocity v_1. At some point, the second mass runs into a spring that is attached rigidly to mass m_2 as shown. The spring constant is a known $k=m_1 g/8$ nt/m.



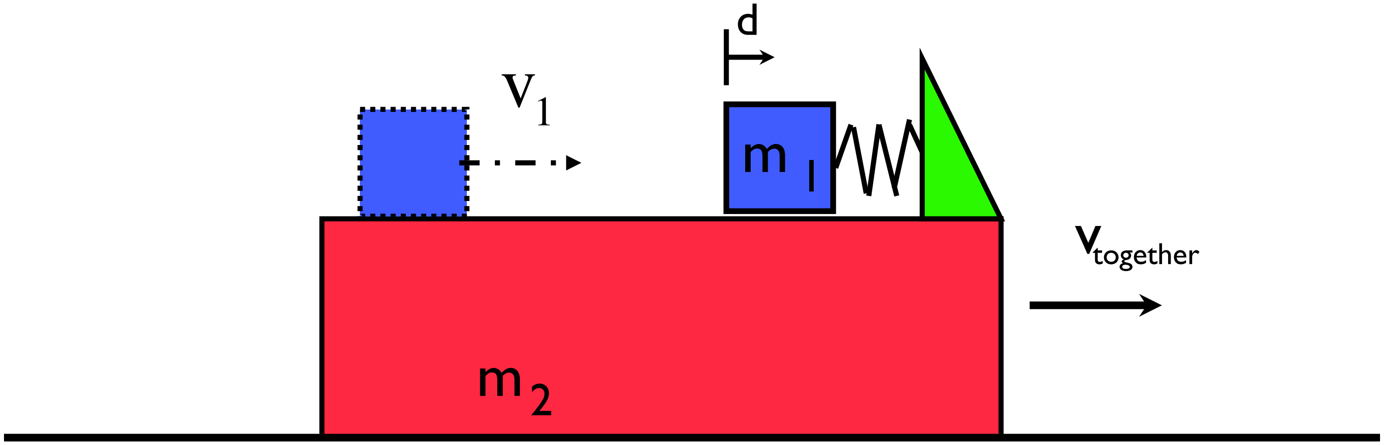


Let's assume $m_2 = 5m_1$ and v_1 is known and equal to 3 m/s. How fast will the two blocks be moving when the spring is depressed its maximum distance d?

Where is energy conserved?

Where is momentum conserved?

Write out the equation(s) needed to determine this velocity.

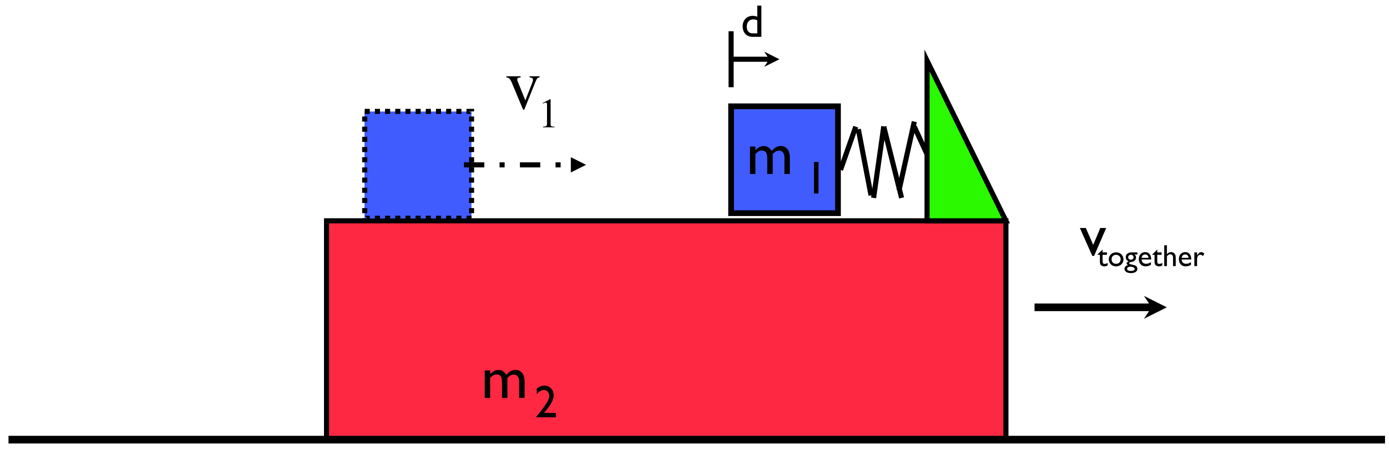


Let's assume $m_2 = 5m_1$ and v_1 is known and equal to 3 m/s. How fast will the two blocks be moving when the spring is depressed its maximum distance d?

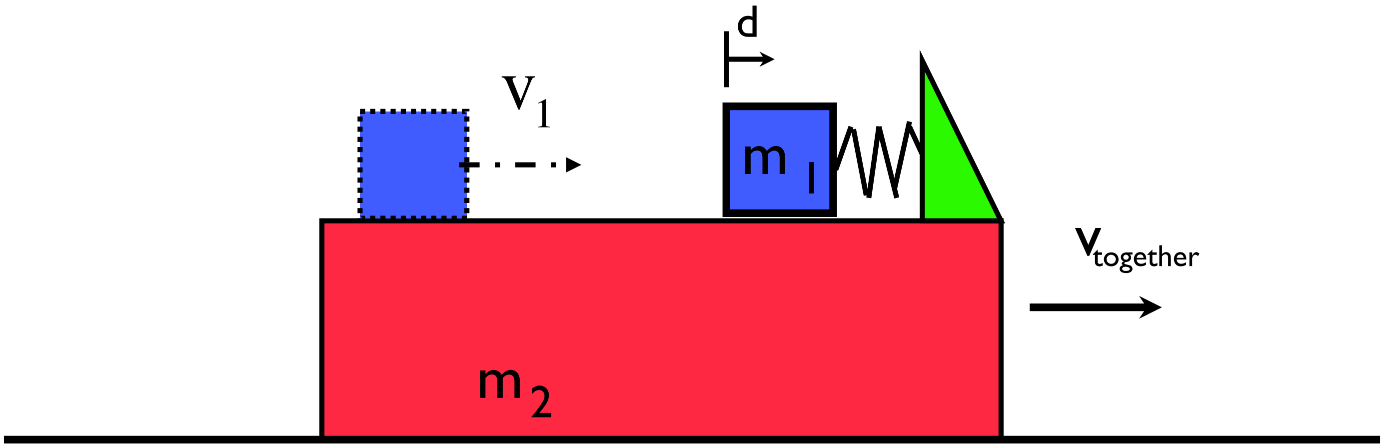
Where is energy conserved? (after the collision)

Where is momentum conserved? (through the collision)

Write out the equation(s) needed to determine this velocity.



d
V 1
m 1
v together
m 2



When the spring is totally depressed, both bodies will momentarily be moving as one. Energy will NOT be conserved through the collision but momentum will be, so we can write:

$$m_1 v_1 = (m_1 + m_2) v_{together}$$

$$\Rightarrow \quad m_1 v_1 = (m_1 + 5m_1) v_{together}$$

$$\Rightarrow \quad v_{together} = \frac{1}{6} v_1$$

$$\Rightarrow \quad v_{together} = \frac{1}{6}(3 \text{ m/s})$$

$$\Rightarrow \quad v_{together} = .5 \text{ m/s}$$

Let's assume there is $3kd^2$ worth of energy is lost during the collision of the top mass with the spring, where "k" is the spring constant and "d" the spring's maximum deflection. By how much is the spring compressed at maximum deflection?

As we know how much energy was lost during the "collision," we can use the modified *conservation of energy* relationship through the collision:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_1 v_1^{\,2} + 0 + (\text{-}3kd^2) = \left(\frac{1}{2}m_1 v_{together}^{\,2} + \frac{1}{2}m_2 v_{together}^{\,2}\right) + \frac{1}{2}kd^2$$

Multiplying everything by 2 to get rid of the 1/2's and messing with the masses, we get:

$$m_1 v_1^{\,2} + (\text{-}6kd^2) = (m_1 + m_2)v_{together}^{\,2} + kd^2$$

$$\Rightarrow\ m_1 v_1^{\,2} \text{-} 6\left[m_1 g/8\right](d^2) = (m_1 + 5m_1)(.5)^2 + \left[m_1 g/8\right]d^2$$

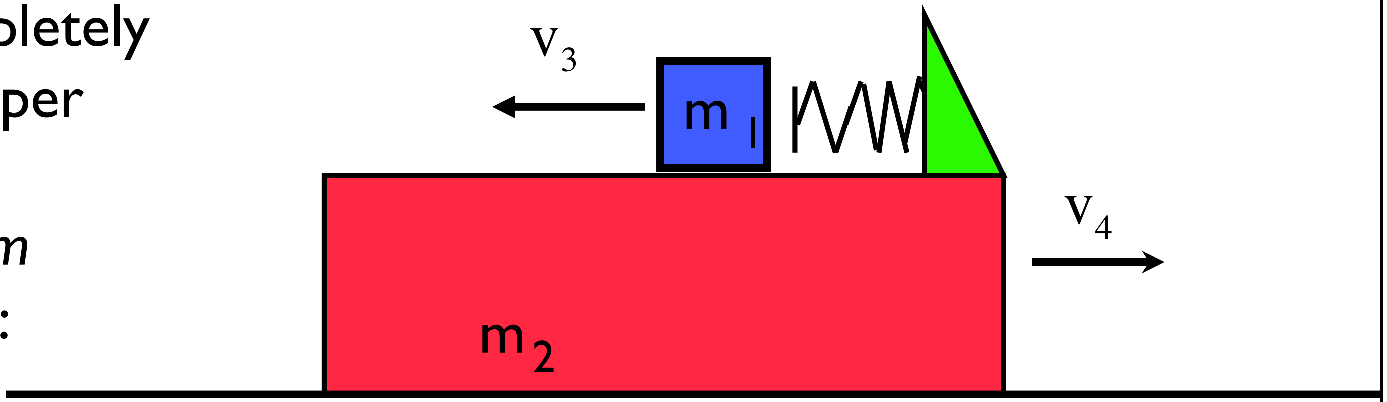
$$\Rightarrow\ m_1(3m/s)^2 - 6\left[m_1(9.8m/s^2)/8\right](d^2) = 6m_1(.5)^2 + \left[m_1(9.8m/s^2)/8\right](d^2)$$

$$\Rightarrow\ 9 - 7.35d^2 = 1.5 + 1.125d^2$$

$$\Rightarrow\ d = .94\ m$$

How fast will the two blocks be moving after the spring has shot the top block back toward the left?

When the spring is completely depressed, it fires the upper mass back to the left. *Conservation of momentum* through that firing yields:



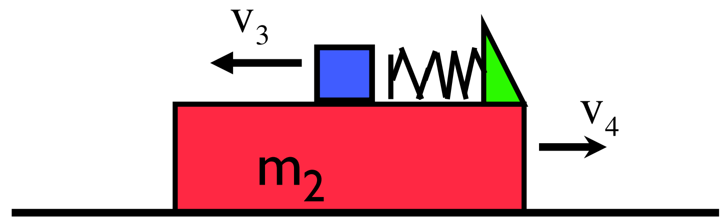
$$(m_1 + m_2)v_{together} = -m_1v_3 + m_2v_4$$

$$\Rightarrow \quad (m_1 + 5m_1)(.5) = -m_1v_3 + 5m_1v_4$$

$$\Rightarrow \quad 3m_1 = -m_1v_3 + 5m_1v_4$$

$$\Rightarrow \quad 3 = -v_3 + 5v_4$$

Conservation of energy through that firing yields:



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\cancel{\tfrac{1}{2}}(m_1 + m_2)(v_{together})^2 + \cancel{\tfrac{1}{2}}kd^2 + 0 = \left(\cancel{\tfrac{1}{2}}m_1 v_3{}^2 + \cancel{\tfrac{1}{2}}m_2 v_4{}^2\right) + 0$$

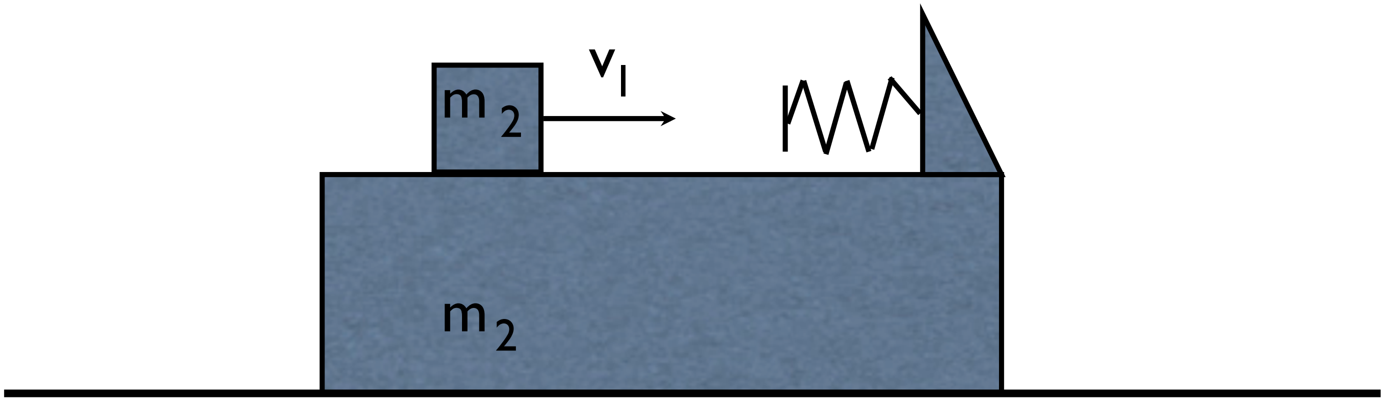
$$\Rightarrow \quad (m_1 + 5m_1)(.5)^2 + kd^2 = m_1 v_3{}^2 + 5m_1 v_4{}^2$$

$$\Rightarrow \quad (6m_1)(.25) + [m_1 g / 8]d^2 = m_1 v_3{}^2 + 5m_1 v_4{}^2$$

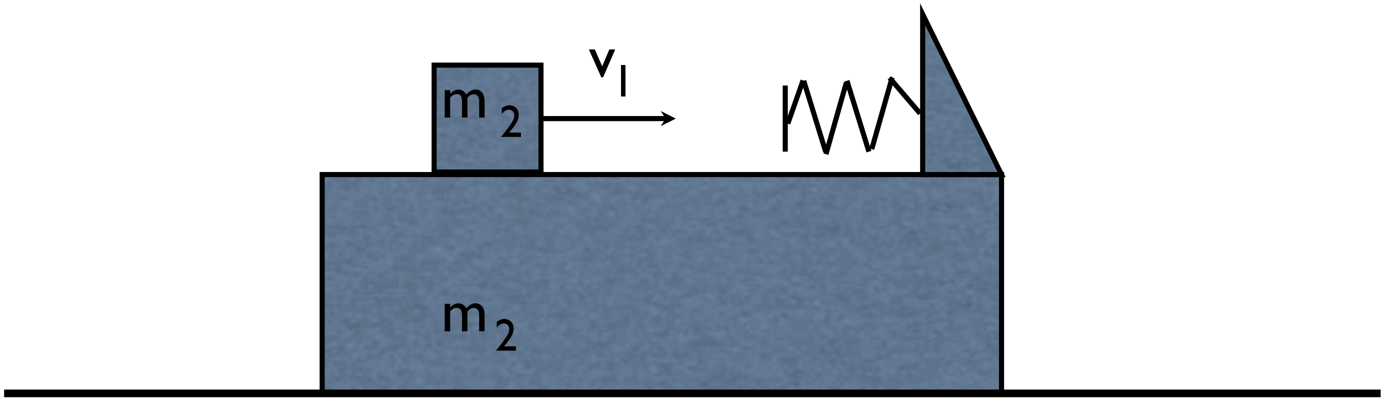
$$\Rightarrow \quad 1.5 + [(9.8)/8](.94)^2 = v_3{}^2 + 5v_4{}^2$$

$$\Rightarrow \quad 2.58 = v_3{}^2 + 5v_4{}^2$$

Algebraically combining the momentum and energy equations will allow you to solve for the two unknowns.



Different Twist: Instead of assuming that $3kd^2$'s worth of energy was lost during the spring collision, assume 2/3 of the *kinetic energy* was lost during the collision. What equations would you use to determine a value for the maximum spring depression *d* in that case?



Assuming that 2/3 of the kinetic energy was lost during the collision, what equations would you use to determine a value for the maximum spring depression *d*?

$$\sum p_{1,x} + \sum F_x \Delta t = \sum p_{2,x}$$

$$m_1 v_1 + 0 = (m_1 + m_2) v_{together}$$

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2} m_1 (v_1)^2 + 0 + \left[-\frac{2}{3} \left(\frac{1}{2} m_1 (v_1)^2 \right) \right] = \left(\frac{1}{2} (m_1 + m_2)(v_{together})^2 \right) + \frac{1}{2} k d^2$$